Fast Phase Diversity Wavefront Sensing for Mirror Control

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ABSTRACT
We show with simulation experiments that closed-loop phase-diversity can be used without numerical guard-bands for wavefront sensing of low-order wavefronts from extended objects using broad-band filters. This may allow real-time correction at high bandwidth for certain applications. We also present a proper maximum likelihood treatment of Shack–Hartman data, which includes an imaging model to extract curvature information from the lenslet images. We demonstrate by simple simulations that this approach should allow higher-order wavefront information to be extracted than with with traditional Shack–Hartmann wavefront sensing for a given number of lenslets.

Keywords: Phase diversity, wavefront sensing, adaptive optics, Shack–Hartmann

1. INTRODUCTION
Wavefront sensing methods for real-time control of adaptive optics (AO) systems need to combine accuracy (sensitivity), large capture range, low cross-talk with unwanted aberrations and speed with ease of alignment and operation. For certain applications, such as with solar telescopes, wavefront sensors operating on extended objects are needed.

In recent years, the most common wavefront sensors for AO systems have been Shack–Hartmann (SH) wavefront sensors and curvature sensors.\(^1\) Both types of sensors have in common that the computational effort required to obtain estimates of the wavefront is modest. SH wavefront sensors estimate the mean gradient of the wavefront over each sub-image formed by a lenslet array. A disadvantage of this is that the mean gradient of the wavefront over the subapertures increases with increasing order of the Zernike aberrations, giving large contributions to the relative movements of the corresponding sub-images. This leads to strong cross-talk with higher-order aberrations. With \(N\) lenslets, no more than approximately \(N/2\) Zernike modes can be estimated.\(^2\) SH wavefront sensing works with extended objects and has been used with solar telescopes.\(^3\) A fundamental limitation for such applications is that the number of lenslets is restricted by the requirement that each sub-image must resolve solar fine structure. For SH wavefront sensors, calibration of the relative positions of the SH sub-images is required and the stability of such calibrations can constitute a problem. Curvature sensors have been successfully operated with AO systems on the Canada-France-Hawaii Telescope (CFHT) and other telescopes. They have the advantage of requiring virtually no computations and calibration, but their possible use for wavefront sensing on extended objects appears doubtful and yet needs to be demonstrated. In addition, it appears that the sensitivity of these types of sensors is poor for wavefront tilt, which is the dominating atmospheric aberration.

Phase diversity (PD)\(^1\)–\(^9\) wavefront sensors are very simple optically and require only a simultaneous focused and defocused image. PD sensors are also easy to calibrate. However, PD sensors that depend on an imaging model involving convolutions, which are usually implemented using FFTs, and on a maximum likelihood (ML) estimate of
the wavefront, are computationally demanding. In this paper, we draw attention to the fact that present workstations approach sufficient speed to allow the implementation of such compute-heavy analysis of wavefront data for real-time control of AO systems. For example, a $16 \times 16$ real–complex FFT can be executed in 25 $\mu$s on a 550 MHz work station. It is therefore appropriate to investigate the possible implementation of PD methods which are based on computationally heavy methods but which may help to overcome some limitations of existing wavefront sensing methods by allowing more accurate wavefront information to be extracted from the data. We emphasize that e.g. conventional analysis of SH wavefront data discards information about wavefront curvature over the lenslets and that it is only by more detailed modeling that all relevant information on the wavefronts can be extracted, which in principle should be made down to the noise level of the data.

SH data have been considered for ML analysis earlier in a formulation that requires also a high-resolution CCD, synchronized with the SH CCD. In this formulation, the lenslet images collectively are regarded as a single phase diverse channel. The lenslet array is then viewed as a single phase screen that produces a number of separate sub-images.

At Lockheed-Martin Missiles & Space (LMMS), PD laboratory experiments have been performed where stable corrections down to 1/20 of a wave RMS were obtained. For these experiments an implementation based on the General Regression Neural Net (GRNN), that can also operate with wide-band data, was used. For a small number of modes, the processing speed is dominated by the calculation of the PD metric that is input by the GRNN. However, the size of a training set can grow very rapidly with the number of controlled parameters. This may cause problems with some applications. Another disadvantage with neural net approaches is that the net has to be trained, which can be a difficult task in a system where seeing is the primary source of aberrations. PD implementations based on models of the imaging system and joint ML estimation of the wavefront and the object are slower with extended scenes, since that usually involves evaluation of the error metric in image space, using a “guard band” of one form or another to avoid the errors that come from performing convolution on finite subfields. The reason for the reduced performance is that this leads to additional inverse Fourier transforms, proportional to the number of corrected modes. Also, when an iterative optimization method is used, accurate modeling of the imaging system can be an additional burden in the wide-band case, when derivatives for several wavelengths have to be calculated for each iteration.

We are also in the process of developing a ML formulation of SH data which does not require a synchronized high-resolution CCD. In this formulation, the sub-images are considered as different channels from a single wavefront, and an imaging model is combined with an ML approach to derive that wavefront.

In this paper we present two separate simulation experiments that each address some of the problems and requirements for wavefront sensing using an extended object. The experiments are designed within the framework of a solar telescope, although they should be relevant also for other telescopes that are used with extended objects. Our treatment of SH data may also be of interest for AO wavefront sensing with point-like objects and for post-processing of high-resolution images recorded with SH wavefront sensors.

Section 2 is an overview of the proper ML treatment of image data for wavefront sensing with an unknown object. In Section 3 we present a wide-band simulation experiment that shows that wavefront sensing from extended objects can be used with ML PD to develop algorithms suitable for real-time applications. In Section 4 we develop the ML treatment into an amplitude diversity formulation for SH data, similar to PD, and show by simulations that it can extract high-order wavefront information not used by traditional SH wavefront sensing. Some conclusions of these experiments are drawn in Section 5.

2. MAXIMUM LIKELIHOOD ESTIMATION OF WAVEFRONTS FROM IMAGE DATA

2.1. Data model

PD is one example of a broad class of existing and possible wavefront estimation methods. A general data collection model that incorporates diversity in phase, amplitude, and wavelength is given in Ref. 10. It describes the data
sets used with different methods such as Phase Diverse Speckle, SH wavefront sensing, Speckle interferometry, blind deconvolution, etc. It is pointed out that the common framework suggests a unified approach using estimation-theoretic methods, rather than relying on ad hoc estimators. The idea is to collect images of a common object through at least two different image channels and jointly find consistent estimates of the object and model parameters that describe the image formation in the different channels, including the unknown phase.

Here, our imaging model (within an isoplanatic patch) is as follows: The Fourier transform, $D_k$, of an image $\mathbf{d}_k$, in image channel number $k \in \{1, 2, \ldots, K\}$ of an object $\mathbf{F} = \mathcal{F}\{\mathbf{f}\}$, is formed according to

$$D_k(u) = \mathcal{N}\left\{ \int_\lambda F(u; \lambda) S_k(u; \lambda) r_k(\lambda) d\lambda \right\},$$

(1)

where $u$ is the spatial frequency coordinate on the detector and $\lambda$ is the wavelength. $S_k$ is the short-exposure optical transfer function (OTF) in channel $k$, $r_k(\lambda)$ is the combined spectral responsivity of the atmosphere, the detector and any optics and filters in the image channel. $\mathcal{N}$ is a noise operator. This is a general enough description of image formation for our purposes. Specifically, it incorporates broadband light through the integration over $\lambda$, as well as diversity in phase and/or amplitude through the factor $S_k(u; \lambda)$, which is the normalized auto-correlation of the aperture function, $P_k$.

$$S_k(u; \lambda) = (P_k \circ P_k)(u; \lambda),$$

(2)

where

$$P_k(u; \lambda) = A_k(u; \lambda) e^{i(\phi(u; \lambda) + \theta_k(u; \lambda))}.$$  

(3)

A changing turbulent atmosphere, as well as any instrumental aberrations, are represented by the unknown phase $\phi$ and there is possible diversity in the phase term $\theta_k$ as well as in the amplitude $A_k$.

Under the assumption that the spatial dependence of the object is separable from the wavelength dependence, $f(x; \lambda) = f(x)g(\lambda)$, wide-band data can be represented by summation of suitably sampled terms.\textsuperscript{10} We can then use a spectral average of the transfer function,

$$S_k(u) = \frac{1}{\Lambda} \sum_\lambda S_k(u; \lambda) g(\lambda) r_k(\lambda).$$

(4)

In this case (as well as for narrow band data) and under the assumption of additive noise, an adequate (and certainly preferable from a computational point of view) model for low-contrast targets such as solar granulation is obtained by reducing Eq. (1) to

$$D_k(u) = F(u) S_k(u) + N_k(u).$$

(5)

For a Gaussian additive noise model, the ML estimate of the object and the wavefront(s) corresponds to a least-squares fit to the data such that the total RMS difference between the images observed in the different channels and those assumed by the imaging model, is minimized. A detailed derivation is given in Ref. 6. In essence one minimizes an error metric of the form

$$L = \sum_u \sum_k^K |D_k - F S_k|^2.$$  

(6)

The optimum object estimate can be expressed in $D_k$ and $S_k$ and can therefore be solved for implicitly. The metric can be rewritten as

$$L = \sum_u \sum_k^K |D_k|^2 - \sum_u \sum_k^K \frac{|D_k S_k|^2}{\sum_k^K |S_k|^2}.$$  

(7)

In this equation the first term is simply a constant which can be ignored in the optimization process.

\textsuperscript{*} We usually multiply the images, $d_k$, with an apodization window function prior to transforming.
2.2. Linearized solution method for $K = 2$

The metric in Eq. (7) can be optimized with standard non-linear optimization methods. For post-processing with two diversity channels, our approach has been to linearize the problem, so that it converges to an ML estimate in just a few iterations by repeatedly solving a system of equations. The greatest correction towards the optimum is usually taken in the first step, as is expected by a Gauss–Newton method, which should give a faster initial convergence than both Newton and gradient methods. As the first iteration, starting from a zero initial solution, is computationally much cheaper than the following, this is an attractive property in a method for closed loop wavefront sensing.

For two diversity channels ($K = 2$) and one realization of the unknown turbulence ($J = 1$), the error metric can be written as

$$L = \sum_u |E|^2,$$

where the Fourier-domain error function is defined as

$$E = H(D_2\tilde{R}_1 - D_1\tilde{R}_2),$$

and $\tilde{}$ stands for an estimated quantity, $H$ is a low-pass noise filter, and

$$R_k = S_k(|S_1|^2 + \gamma|S_2|^2)^{-1/2}.$$

The unknown wavefront $\phi$ is expanded in a set of basis functions, $\{\psi_m\}$ (e.g. Zernike polynomials),

$$\hat{\phi} = \sum_m \alpha_m \psi_m.$$

Through Eqs. (2), (3), and (11), it is clear that $L$ is a non-linear function of the wavefront expansion parameters, $\{\alpha_m\}$. The minimum has to be found iteratively from an initial estimate, usually $\phi \equiv 0$. Changes in $E$ are approximated with

$$\delta E \approx \sum_m \frac{\partial E}{\partial \alpha_m} \delta \alpha_m,$$

and corrections to the coefficients, $\delta \alpha_m$, such that the minimum of $L$ is found in the next iteration are sought. Note that

$$\frac{\partial E}{\partial \alpha_m} = H \left( D_2 \frac{\partial R_2}{\partial \alpha_m} - D_1 \frac{\partial R_1}{\partial \alpha_m} \right)$$

where $\partial R_k/\partial \alpha_m$ can be pre-calculated for $\phi = 0$, which reduces the computational burden significantly. This is so because the linearization results in a matrix equation of the type

$$A \cdot \delta \alpha + b = 0,$$

where

$$A = \begin{pmatrix}
\left( \frac{\partial E}{\partial \alpha_1} | \frac{\partial E}{\partial \alpha_1} \right) & \cdots & \left( \frac{\partial E}{\partial \alpha_1} | \frac{\partial E}{\partial \alpha_M} \right) \\
\vdots & \ddots & \vdots \\
\left( \frac{\partial E}{\partial \alpha_M} | \frac{\partial E}{\partial \alpha_1} \right) & \cdots & \left( \frac{\partial E}{\partial \alpha_M} | \frac{\partial E}{\partial \alpha_M} \right)
\end{pmatrix}; \quad \delta \alpha = \begin{pmatrix}
\delta \alpha_1 \\
\vdots \\
\delta \alpha_M
\end{pmatrix}; \quad b = \begin{pmatrix}
\frac{\partial E}{\partial \alpha_1} | E \\
\vdots \\
\frac{\partial E}{\partial \alpha_M} | E
\end{pmatrix},$$

and the inner-product is defined mathematically as

$$(A \mid B) = \sum_u AB^*.$$
Numerically, however, due to FFT wrap-around errors in the evaluation of the inner products, we have used an image space inner-product, that allows us to restrict the summation to areas in the center of the FOV, that are less affected by these errors. We therefore define

\[
(A \mid B) = \sum_{x \in \mathcal{X}} \mathcal{F}^{-1}\{A\}(x) \cdot \mathcal{F}^{-1}\{B\}(x)
\]

where \(\mathcal{F}^{-1}\) is the inverse Fourier transform, and \(\mathcal{X}\) is a centered field of view, surrounded by a guard band where large wrap-around errors do not influence the result. Parseval’s identity and related theorems assure that with no guard band, Eq. (17) is equivalent with Eq. (16). However, the wrap-around errors can be avoided with a proper choice of \(\mathcal{X}\). Using Eq. (17) could be very time-consuming, because not only the \(E\) matrix but also its derivatives have to be inverse Fourier transformed, but it is necessary for convergence when an accurate solution is wanted for large wavefronts.

A version of this algorithm has been successfully used in laboratory closed-loop experiments at LMMS.

3. FREQUENCY SPACE PHASE-DIVERSITY

Using the inner-product in Eq. (17) is computationally expensive for high bandwidth closed loop applications. It is usually needed with our PD algorithm because finite-FOV images are convolved with wide PSFs corresponding to large wavefronts. The wider the PSF, the more information is carried across array boundaries, so the influence of these errors should be smaller for almost corrected systems when the PSFs are more concentrated. For AO control, when only an approximate solution is needed from each image pair because the next estimate will come from a corrected image, the need for Eq. (17) should be less than for applications where accurate estimates of large wavefronts are needed.

We have performed closed-loop simulations that demonstrate that Eq. (16) can be used for wavefront sensing with an extended object, making the algorithm clearly dominated by the inner-products. For each of the ten cases presented, a random wavefront was used to generate OTFs corresponding to two image channels with a 1-wave peak-to-peak difference in focus. The input wavefronts contain contributions from Zernike aberrations 4–8 with random amplitudes corresponding to Kolmogorov statistics and scaled to a fixed RMS of 350 nm. We simulated wide-band data by creating OTFs as sums of \(S_k(u; \lambda)\) for \(\lambda \in \{500, 520, \ldots 800\} \text{ nm}\), see Eq. (4). Gaussian noise with an RMS of about 10% of the RMS contrast of the unaberrated image added to each frame according to Eq. (5).

A single-iteration estimate of the wavefront is generated using Eq. (16) and subtracted from the input wavefront. The residual in each step is used to produce a new simulated image pair. The PD inversions were performed with \(\lambda \in \{550, 650, 750\} \text{ nm}\), a much sparser sampling, corresponding to the realistic scenario when the spectrum is continuous and spectral under-sampling cannot be avoided. Note that using several wavelength samples for the inversions does not increase the processing time in the single-iteration mode, since \(\partial R_k/\partial \phi_m\) are pre-calculated for \(\phi = 0\).

Figure 1 shows image data for one of the cases. Figure 2 shows for all cases the RMS of the residual wavefronts and Strehl ratios as functions of the number of corrections. The RMS = 350 nm for all the initial wavefronts and the converged wavefronts had RMS ≈ 10 nm. The initial Strehl ratios were in the range 4–13% and the converged values were all approximately 99%.

4. SHACK–HARTMANN DATA AND DIVERSITY ANALYSIS

4.1. Maximum Likelihood Shack–Hartmann Wavefront Sensing

A Shack–Hartmann wavefront sensor consists of a lenslet array in a conjugated pupil plane and a CCD that records the resulting array of images. Each lenslet image corresponds to a single object, common to all the lenslet images, which is displaced by local (over the area corresponding to that lenslet) wavefront tilts and degraded by local wavefront curvature. Simulated SH data in Fig. 3 show how tilts and curvature differ for focus and spherical aberrations.
Figure 1. Image data from a frequency-space PD simulated closed-loop experiment. The FOV corresponds to 10.6 arcsec on the sun. The (c) and (d) tiles show images corresponding to an initial wavefront (350 nm RMS) that gave a 4% Strehl ratio and a converged wavefront residual (10 nm RMS) with 99% Strehl, resp.

Figure 2. Output from frequency-space PD simulated closed-loop experiment.

The images in Fig. 3(c) and (d) are identical except for the displacements because the local curvature is the same in all the sub-apertures. This curvature is small, as can be seen by comparison with the diffraction limited data in (a). The spherical aberration images in Fig. 3(e) and (f), however, show local astigmatism in the corner images (3, 5, 7, 9), while there is almost no additional blurring in (2, 4, 6, 8), not even in Fig. 3(f). This is because the local curvature close to the center is small for spherical aberrations. In classical SH wavefront sensing, only the tilt information in the relative image shifts is used, while the information encoded in the blurring is discarded. A more realistic imaging model such as used for PD, should be able to utilize this information. By combining this imaging model with a ML estimate of the aberration coefficients, we expect to minimize the errors in the derived aberration coefficients from random noise in the data.
Figure 3. Simulated SH data. The positions of the 32 × 32-pixel tiles in (b)–(f) correspond to the sub-aperture geometry in (a).
In PD, the OTFs of different imaging channels involve the unknown wavefront plus known phase perturbations, \( \theta_k \), of the aperture function, \( P_k \), see Eq. (3). The different lenslet images in a SH wavefront sensor correspond to sampling different sub-areas of the aperture. This is equivalent to diversity in the amplitude, \( A_k \), of the aperture function in the sense that the amplitude corresponds to different apodization functions, with unit amplitude over the shape of lenslet number \( k \). A better name for this special case of amplitude diversity is aperture diversity, which relates more directly to the fact that we are making measurements of the same wavefront from different sub-apertures across the telescope aperture. As with PD, we could try to jointly determine the unknown object and the unknown wavefront, expanded in some set of basis functions such as the Zernike polynomials, over the full entrance aperture of the telescope. The same coefficients of this wavefront expansion appear for all imaging channels but the wavefronts over the different lenslets differ according to their geometrical location within the aperture.

With this view, the SH wavefront sensor is mathematically equivalent to a PD wave-front sensor in the sense that the same error metric, \( L \) (Eq. (7), valid for Gaussian additive noise), is optimum and gives true ML estimates of the wavefront. Minimizing this error metric should therefore be preferable to methods which first determine the relative registration of the lenslet images using cross-correlation techniques and then, as a second step in the analysis, apply some sort of wavefront fitting procedure to the so-obtained local wavefront tilts in the sense that we expect higher accuracy in the determined wavefront. In particular, we can take into account also fairly strong wavefront curvature over each lenslet.

Because the lenslets are usually much smaller than the whole aperture, the RMS of the wavefront curvature over each lenslet is much smaller than the RMS of the wavefront over the whole aperture. We therefore expect convergence, using the linearized approach, for much poorer seeing conditions than for PD.

### 4.2. The no-curvature case

Low-order wavefronts over the lenslets are typically dominated by large wavefront tilts and small curvatures. It may be interesting to compare the computational effort of the proposed method with a proper ML treatment involving the no-curvature approximation used in classical SH wavefront sensing.

We derive \( L \) for the case where we ignore wavefront curvatures across the lenslets but retain the tilt terms. Then \( S_k = S_a T_k \), where \( S_a \) is the lenslet diffraction-limited transfer function, which is the same for all lenslets and which disappear from the error metric. \( T_k \) are the tilt terms which depend on the wavefront expansion coefficients, \( \alpha_m \), and of course are different for the different lenslets with index \( k \). The error metric in Eq. (7) simply becomes

\[
L = \sum_u \sum_k |D_k|^2 - \sum_u \frac{1}{K} \sum_k D_k T_k^u \quad (18)
\]

where \( K \) is now the number of lenslets and we have used \( |T_k|^2 \equiv 1 \). Minimizing \( L \) therefore corresponds to finding the best relative tilts, or shifting the images in the \( x \) and \( y \) directions, such that the algebraic sum of the lenslet images has the highest RMS contrast.

When in closed-loop with an adaptive mirror, we can assume that the expansion coefficients \( \alpha_m \) remain small such that we can make a Taylor expansion of \( S_k \) in terms of \( \alpha_m \), keeping only the first order terms. This means that we can write

\[
T_k = 1 + \sum_m \frac{\partial T_k}{\partial \alpha_m} \alpha_m 
\]

(19)

where \( \partial T_k/\partial \alpha_m \) are evaluated for the no-tilt case \( T_k \equiv 1 \) for each lenslet \( k \) (equivalent to \( \alpha_m = 0 \) for all \( m \)) and therefore can be pre-calculated and stored in memory. We now obtain

\[
\sum_k D_k T_k^u = \sum_k D_k + \sum_m \alpha_m \sum_k D_k \frac{\partial T_k^u}{\partial \alpha_m} 
\]

(20)
For the particular case of pure wavefront tilts, we can write

\[
\frac{\partial T_k}{\partial \alpha_m} = \ell_{km} u
\]

where \(u\) is the Fourier variable and \(\ell_{km}\) are constants which in general are different for each \(k\) and \(m\). Because of the multiplication of \(D_k\) with \(\ell_{km}\), evaluating the error metric for the simple case of pure tilts requires \(MK\) complex matrix multiplies (last term in Eq. (20)), just as for a more realistic modeling which includes also wavefront curvatures. We conclude that there is no dramatic difference in the computational efforts between a (proper) least-squares fit of the the aberration parameters which assumes that the aberrations can be approximated with local tilts and an approach that assumes small wavefront curvatures over the individual lenslets.

4.3. Simulations

Although it is clear that an ML analysis of the SH wavefront sensor results in an expression that is the same as for a PD sensor, it is by no means obvious that this is an ideal wavefront sensor in the sense of having the required ability to distinguish different aberrations with the same average local tilts but different higher-order curvatures. Therefore we show by some simple tests involving measurements from only two simulated lenslet images, that determination of the wavefront appears possible with higher-order terms using the PD approach than is possible with a classical approach. We use only two image channels (1 and 2 for the experiments shown here, see Fig. 3), since we can then use our existing code that implements the method outlined in Section 2.2.

SH data corresponding to the Swedish Vacuum Solar Telescope (SVST) (focal length 22.35 m, aperture diameter 0.475 m) and a lenslet geometry as shown in Fig. 3(a), were generated using different wavefronts. We did not include noise in these first exploratory simulations because we primarily wanted to demonstrate that there exists curvature information that allow accurate determinations of high-order wavefronts, and did not yet investigate the regime in which the method would be useful. We did tests for critically sampled data, as well as for data that are approximately 10% over- and under-sampled, respectively. The results shown here were obtained with under-sampled data. The images were created on a \(64 \times 64\)-pixel grid and only the center part used as data. The data are of the same type as those shown in Fig. 3.

We did iterative inversions, rather than a closed loop experiment. The inner-products were evaluated over \(20 \times 20\)-pixel regions, \(X\), in image space, while FFTs were performed on \(32 \times 32\)-pixel arrays, allowing for a 6-pixel guard band. We show in Fig. 4 results that demonstrate that it works for more than a wave RMS, where the differential blurring is quite noticeable between channels 1 and 2. The results after one iteration do not look encouraging (although some cases immediately find correct answers), but after six iterations all cases have converged to good solutions. In another experiment we used smaller magnitude astigmatism and fifth order astigmatism aberrations. Here, virtually perfect solutions are found immediately for those data where the input wavefront has the same sign on Zernike coefficients 6 and 12, see Fig. 5(a) and (b). This corresponds to partial tilt cancellation for the two sub-apertures we used. We note that the convergence was slightly slower for the cases with opposite signs in Fig. 5(c) and (d), although the errors are small already after the first iteration.

5. CONCLUSIONS

We have presented possible improvements of two methods for wavefront sensing.

Using the closed-loop frequency-space inner-product saves a number of FFT calculations, compared to earlier experiments with extended scenes. The presented cases worked very well. The converged residuals are small, we did not exhaust the capture range, and wide-band (several hundred nm) data could be handled. However, similar experiments with wavefronts containing more realistic high-order wavefronts only gave one or two good corrections and tended to diverge after that, also for smaller wavefront RMS. Other under-parameterized cases failed in a similar manner. Also, the capture range of this Fourier-space method does not appear to be too great. More work is
Figure 4. Scatter plots input (x-axis) and output (y-axis) Zernike coefficients from simulations. Axes labeled in waves. Wavefronts are combinations of focus and spherical aberrations. The correlation coefficient between true and estimated coefficients is 0.96 after six iterations for focus and spherical both.

needed before this is implemented for real applications. We note, however, that it may be a good alternative in situations where one expects only a small number of rapidly changing wavefront components. It is possible to design monolithic mirrors for space telescopes such that distortions will only have a well-known small number of significant modes. Also, it may be an alternative for piston sensing for multiple aperture telescopes. We have made some simulations for such cases and our experience is that it can work, particularly if the image contrast is high.

The tentative conclusion of the preliminary SH study is that proper modeling of imaging through a lenslet array may be possible, that such a more realistic approach should be more accurate than *ad hoc* cross-correlation methods, and that including wavefront curvature may enable higher order aberrations to be determined than for a conventional approach to analyzing SH lenslet images. Firmer conclusions require simulations involving an array of lenslets, realistic wavefronts and noise and not just two lenslets which are used to derive two aberrations, as in this preliminary study. If this method can be shown to work, then it has the advantage of requiring exactly the same optics as for conventional SH wavefront sensing, thus allowing immediate switching between the methods in software. This may be useful, e.g. for initialization of the solutions, since classical SH has the advantage of not being limited to small wavefront tilts.
Figure 5. Scatter plots input (x-axis) and output (y-axis) wavefronts from simulations. Axes labeled in waves. Wavefronts are combinations of astigmatism and fifth order astigmatism (Zernike coefficients 6 and 12, resp.). Correlation coefficients after one iteration: 0.91 for (a) and (b). 0.93 for (c) and 0.69 for (d). After three iterations, the correlation in (c) increases to 0.94 and in (d) to 0.90.

The computational burden corresponding to this more realistic modeling of SH data may be an order of magnitude larger than for classical cross-correlation techniques. The $MK$ complex multiplies needed to form the derivatives dominate over the $K$ FFTs, but may not be needed if a 1-iteration closed-loop can be set up. In addition, we need on the order of $M^2$ multiplies to evaluate the inner products required to set up the final matrix equation. However, it is not certain that the same number of image channels is optimum with the ML treatment as with classical SH wavefront sensing. Using a smaller number of channels may help the speed. In more ad hoc approaches, which use one of the lenslet images as a reference image relative to which the displacements of the other lenslet images are obtained with cross-correlation techniques, the number of multiplies is greatly reduced, but such an approach should be more influenced by noise and therefore less accurate.

Although our main interest is in future application to real-time control of adaptive optics systems, the results are valid also for post-processing of images obtained through large telescopes and recorded simultaneously with SH data. For such applications, speed of processing is not a concern and the only priority is more accurately restored images.
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